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## “Relative intrusion as a result of protrusion: A mathematical perspective”

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**Abstract**

The aim of this study was to investigate the role of initial incisor axial inclination on the amount of bite opening, with the aid of trigonometry.

The uncontrolled tipping movement of an upper central incisor that had 9 mm crown and 14 mm root lengths was simulated. The centre of rotation was accepted at the middle third level of the root. Simulation was performed according to the 1°, 5°, and 10° protrusion sequences between 90° and 180°. The amounts of relative intrusion to the true horizontal line were calculated by using a trigonometric formula.

The initial axial inclination was an important factor on bite opening. Increased initial axial inclinations cause much more relative intrusion compared with the lower initial values. The initial incisor axial inclination has an effect on the bite opening aspect of protrusion.

**Keywords:** Initial incisor axial inclination, Protrusion, Relative intrusion, Trigonometry

**Introduction**

Deep bite can cause detrimental effects on anterior dental aligning, periodontal status, temporomandibular health, protrusive-laterotrusion jaw movements, and smile aesthetics [1-3]. Although high rates of deep bite are generally seen in low angle profiles, it can exist with almost all types of malocclusions, regardless of the vertical growth pattern [4].

Deep bite correction is a challenging problem for clinicians. Nonsurgical treatment options involve pure incisor intrusion, incisor protrusion, molar extrusion, or a combination of these [5,6]. Several factors, such as gingival display, skeletal pattern, and growth status, can affect the choice. Molar extrusion is an effective method for bite opening in growing patients; however, if growth is completed, this treatment modality is questionable in terms of stability [7]. Furthermore, the extrusion of posterior teeth can cause a clockwise rotation of the mandible, and this may lead to worsening in class II subjects' profiles. Additionally, in the treatment of class II division 2 patients, upper incisor protrusion and intrusion were suggested instead of molar extrusion. It has been reported that one of the causes of upper incisor retroclination in class II division 2 patients is a high lower lip line. Therefore, the treatment modality should include maxillary incisor intrusion, so the pressure of the lower lip on the upper incisors, which was stated as the etiologic factor, can be decreased [8,9]. Namely, conditions requiring incisor protrusion and intrusion without molar extrusion can be experienced in clinical practice.

Various intrusion mechanics, such as reverse-accentuated curve of Spee, three-piece intrusion, utility, and Connecticut intrusion arches, have been introduced [5,10]. In addition to incisor intrusion, incisor flaring and molar extrusion inevitably occur in all these options [11,12]. The main factor affecting the amount of protrusion is the relationship between the location of the point of force application and the center of resistance of the anterior segment. The more anterior the location of the force application point, the more the incisor protrusion [12].

Although incisor protrusion is an unwanted side effect in patients who have normal or proclined axial inclinations, the labial tipping of anterior teeth is a desirable effect of intrusion arches in patients who have retroclined axial inclinations, such as in class II division 2 malocclusion.

Retroclination of the upper incisors, which can negatively affect the smile aesthetic, is already a malocclusion and must be treated appropriately. Schudy stated that an increased interincisal

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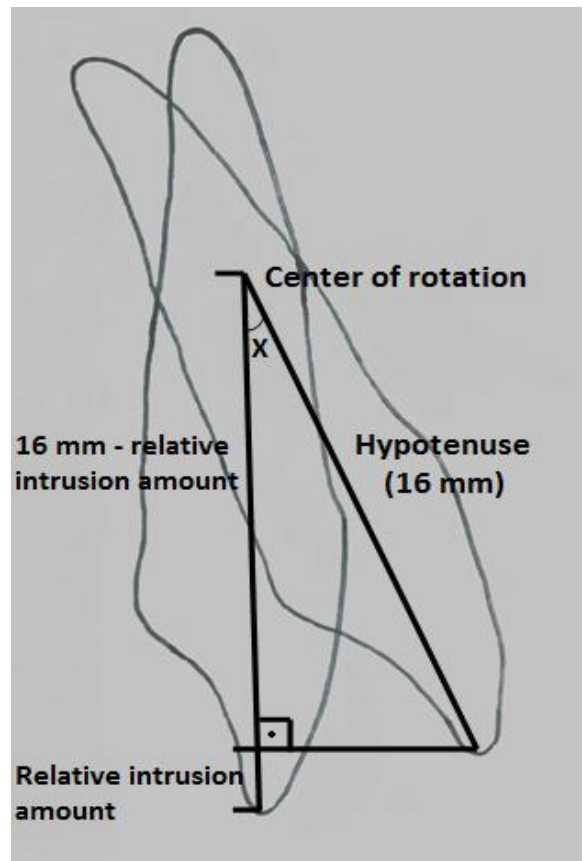
angle is a causative factor in excessive overbite, and correction of this angle should be one of the treatment goals [13]. Furthermore, posttreatment interincisal angle is an important factor in maintaining successful treatment results and avoiding a relapse [14,15]. Protrusion has a remarkable contribution to bite opening as well, with the aid of a change in vertical incisal edge position. Consequently, incisor flaring combined with intrusion was presented as a treatment method in correcting the different malocclusions [6].

Incisor protrusion can be achieved by using various mechanics. However, we clinically observed that the same amount of protrusion does not always result in the same amount of bite opening. In this study, by using the mathematical perspective, we aim to evaluate the influence of initial upper incisor axial inclination on bite opening.

**2. Material and methods**

Because of the theoretical structure of this study, there was no need for ethics approval. In the present study, trigonometry was used.

An upper central tooth with mean dimensional values was hypothetically constructed. The length of this tooth, which consisted of a 9 mm crown and 14 mm root components, was 23 mm. When force was applied at the level of the incisal edge, an uncontrolled tipping movement was caused, and the center of rotation was accepted at the middle third level of the root. As a result of these assumptions, when an uncontrolled tipping movement occurred, a right-angle triangle was obtained where one of the corners was located at the center of rotation and had a 16 mm length of the hypotenuse (Fig 1).



**Fig 1:** Illustration of the simulated tooth

In this study, cosine, which is one of the trigonometric functions, was used for determining the relationship between the initial axial inclination and the amount of relative intrusion. The values of cosine are presented in Table 1.

**Table 1:** Values of cosine

Angle(°)	Value of Cosine	Angle(°)	Value of Cosine	Angle(°)	Value of Cosine	Angle(°)	Value of Cosine	Angle(°)	Value of Cosine
1	0.9998	19	0.9455	37	0.7986	55	0.5736	73	0.2924
2	0.9994	20	0.9397	38	0.7880	56	0.5592	74	0.2756
3	0.9986	21	0.9336	39	0.7772	57	0.5446	75	0.2588
4	0.9976	22	0.9272	40	0.7660	58	0.5299	76	0.2419
5	0.9962	23	0.9205	41	0.7547	59	0.5150	77	0.2249
6	0.9945	24	0.9135	42	0.7431	60	0.5000	78	0.2079
7	0.9926	25	0.9063	43	0.7314	61	0.4848	79	0.1908
8	0.9903	26	0.8988	44	0.7193	62	0.4695	80	0.1736
9	0.9877	27	0.8910	45	0.7071	63	0.4540	81	0.1564
10	0.9848	28	0.8829	46	0.6947	64	0.4384	82	0.1392
11	0.9816	29	0.8746	47	0.6820	65	0.4226	83	0.1219
12	0.9781	30	0.8660	48	0.6691	66	0.4067	84	0.1045
13	0.9744	31	0.8571	49	0.6561	67	0.3907	85	0.0872
14	0.9703	32	0.8480	50	0.6428	68	0.3746	86	0.0698
15	0.9659	33	0.8387	51	0.6293	69	0.3584	87	0.0523
16	0.9613	34	0.8290	52	0.6157	70	0.3420	88	0.0349
17	0.9563	35	0.8191	53	0.6018	71	0.3256	89	0.0174
18	0.9511	36	0.8090	54	0.5878	72	0.3090	90	0.0000

**2.1 Description of used method**

A tooth with a 16 mm length from the center of rotation to the incisal edge was positioned at 90° to the true horizontal line. Protrusion sequences of 1°, 5°, and 10° were simulated from 90° to 180° to the true horizontal, and the amounts of relative intrusion to the true horizontal line were calculated by using

cosine values. For example, when the tooth with a 120° initial axial inclination to the true horizontal line protruded 5°, the following calculation was used:

$$16 \times \cos 30^\circ - 16 \times \cos 35^\circ = 16 (\cos 30^\circ - \cos 35^\circ)$$

A schematic representation of the used method in this study is presented in fig 2.

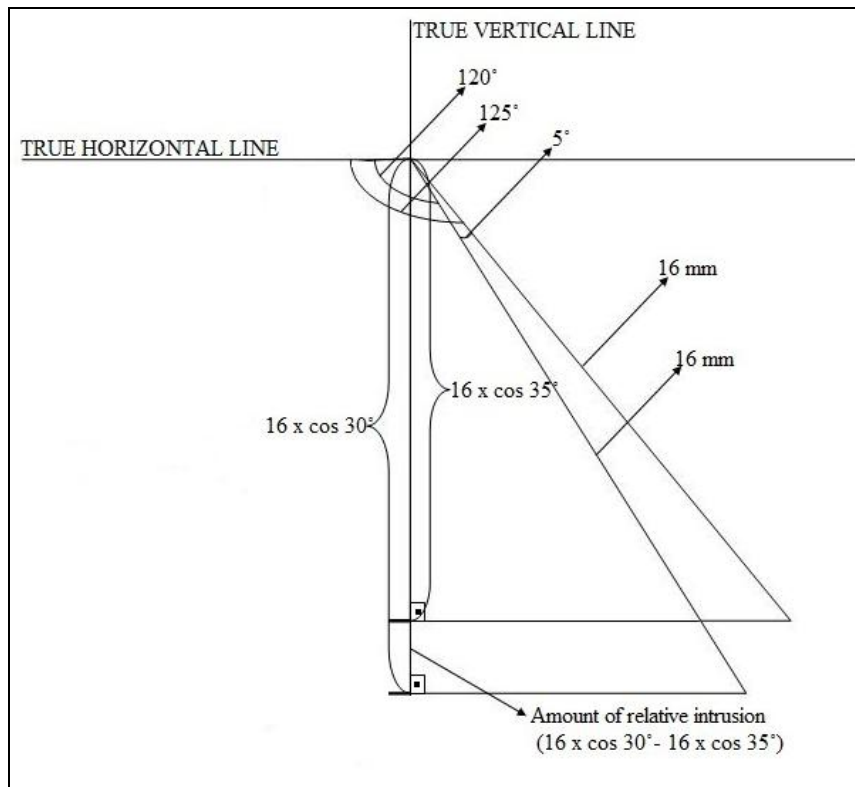


Fig 2: Schematic drawings of the calculation method

3. Results

The amounts of relative intrusion which occurred at each of 1

degree changes of axial inclination to the true horizontal line were calculated and tabulated (Table 2 and Fig 3).

Table 2: The amounts of relative intrusion which occurred at each of 1 degree changes of axial inclination to the true horizontal line

Localization of axial inclination change (°)	Amount of relative intrusion (mm)	Localization of axial inclination change (°)	Amount of relative intrusion (mm)	Localization of axial inclination change (°)	Amount of relative intrusion (mm)
90→91	0.0032	120→121	0.1424	150→151	0.2432
91→92	0.0064	121→122	0.1456	151→152	0.2448
92→93	0.0128	122→123	0.1488	152→153	0.2480
93→94	0.0160	123→124	0.1552	153→154	0.2496
94→95	0.0224	124→125	0.1584	154→155	0.2528
95→96	0.0272	125→126	0.1616	155→156	0.2544
96→97	0.0304	126→127	0.1664	156→157	0.2560
97→98	0.0368	127→128	0.1696	157→158	0.2576
98→99	0.0416	128→129	0.1728	158→159	0.2592
99→100	0.0464	129→130	0.1792	159→160	0.2624
100→101	0.0512	130→131	0.1808	160→161	0.2624
101→102	0.0560	131→132	0.1856	161→162	0.2656
102→103	0.0592	132→133	0.1872	162→163	0.2656
103→104	0.0656	133→134	0.1936	163→164	0.2688
104→105	0.0704	134→135	0.1952	164→165	0.2688
105→106	0.0736	135→136	0.1984	165→166	0.2704
106→107	0.0800	136→137	0.2032	166→167	0.2720
107→108	0.0832	137→138	0.2064	167→168	0.2720
108→109	0.0896	138→139	0.2080	168→169	0.2736
109→110	0.0928	139→140	0.2128	169→170	0.2752
110→111	0.0976	140→141	0.2160	170→171	0.2752
111→112	0.1024	141→142	0.2176	171→172	0.2752
112→113	0.1072	142→143	0.2224	172→173	0.2768
113→114	0.1120	143→144	0.2240	173→174	0.2784
114→115	0.1152	144→145	0.2272	174→175	0.2768
115→116	0.1200	145→146	0.2304	175→176	0.2784
116→117	0.1248	146→147	0.2336	176→177	0.2800
117→118	0.1296	147→148	0.2352	177→178	0.2784
118→119	0.1328	148→149	0.2384	178→179	0.2800
119→120	0.1376	149→150	0.2400	179→180	0.2784

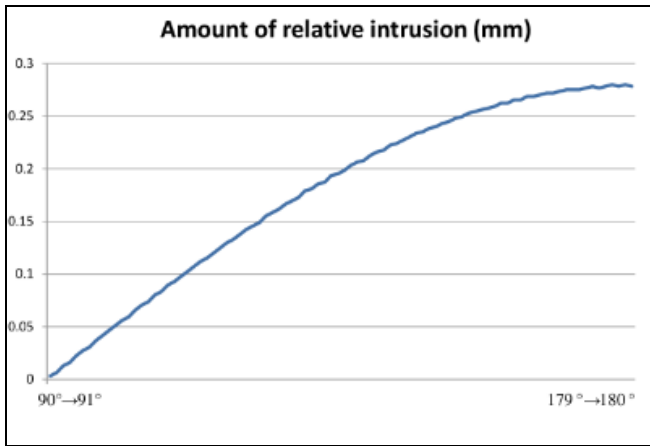


Fig 3: Relative intrusion at each of 1 degree changes

The amounts of relative intrusions that occurred at each 5°

change of axial inclination to the true horizontal line were calculated and tabulated (Table 3 and Fig 4).

Table 3: The amount of relative intrusions that occurred at each of 5° changes of axial inclination to the true horizontal line

Localization of axial inclination change (°)	Amount of relative intrusion (mm)	Localization of axial inclination change (°)	Amount of relative intrusion (mm)
90→95	0.0608	135→140	1.0288
95→100	0.1824	140→145	1.1072
100→105	0.3024	145→150	1.1776
105→110	0.4192	150→155	1.2384
110→115	0.5344	155→160	1.2896
115→120	0.6448	160→165	1.3312
120→125	0.7504	165→170	1.3632
125→130	0.8496	170→175	1.3824
130→135	0.9424	175→180	1.3952

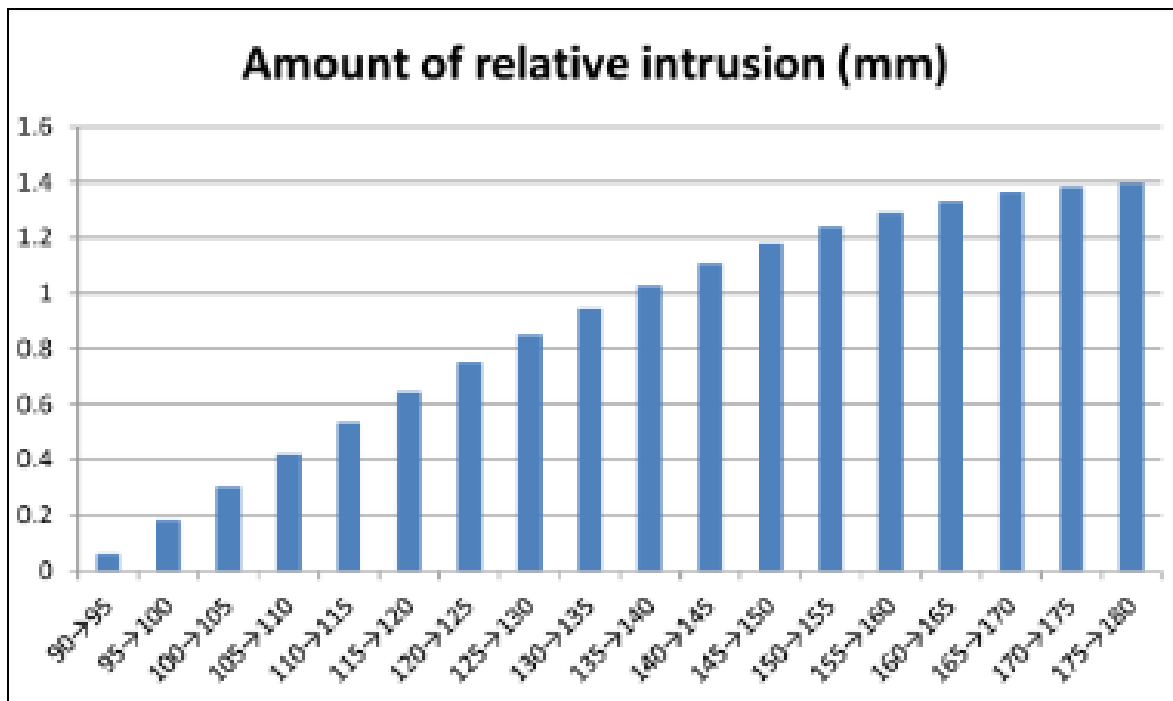


Fig 4: Relative intrusion at each of 5 degree changes

The amounts of relative intrusion which occurred at each of 10 degree changes of axial inclination to the true horizontal line were calculated and tabulated (Table 4 and Fig 5).

Table 4: The amounts of relative intrusion which occurred at each of 10 degree changes of axial inclination to the true horizontal line

Localization of axial inclination change (°)	Amount of relative intrusion (mm)
90→100	0.2432
100→110	0.7216
110→120	1.1792
120→130	1.6000
130→140	1.9712
140→150	2.2848
150→160	2.5280
160→170	2.6944
170→180	2.7776

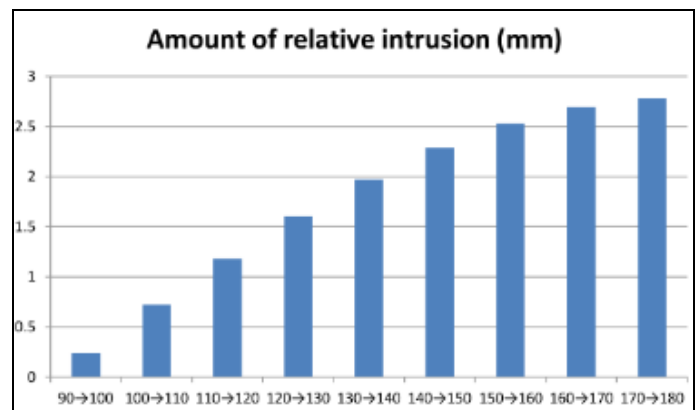


Fig 5: Relative intrusion at each 10 degree changes

#### 4. Discussion

Trigonometry, which observes the relationships between the lengths and angles of triangles, has been known since ancient times. It consists of trigonometric functions, such as sine (sin), cosine (cos), tangent (tan), cosecant (cosec), secant

(sec), and cotangent (cot), which present these relationships. Today, it has been used in different fields of science, such as forensic medicine, bioengineering, oral implantology, sports sciences, astronomy, and geometry [16-19]. However, to date, the use of trigonometry has been neglected in the field of orthodontics, except in the analysis of force vectors [20].

Whether the situation is a treatment goal or an unwanted side effect, incisor protrusion inevitably occurs in many orthodontic treatments. When protrusion is expected, clinicians should be aware of the positive and negative implications of this movement. Although protrusion is a beneficial condition in deep bite cases, it can be an unwanted side effect in patients who have retroclined incisors with a normal or decreased overbite. Therefore, the interaction between the sagittal and vertical movements must be well evaluated. However, the influence of protrusion on bite opening has been neglected and we could not reach the formula about the relationship between protrusion and bite opening. In this study, in addition to the formula, we aimed to show that the amount of bite opening as a result of protrusion can be affected by the initial axial inclination.

In this study, different amounts of protrusion of the upper incisor were simulated and the amounts of relative intrusion to the true horizontal line were evaluated. The total length of the upper incisor was considered to be 23 mm, and the distance between the center of rotation, which occurred during uncontrolled tipping and incisal edge, was accepted as 16 mm. Compared to the literature, these assumptions are reasonable [21]. We aware that the exact location of the center of resistance and the center of rotation for a tooth is not easily identified. The center of resistance of the tooth depends on the intrinsic factors such as the root length, morphology and the status of periodontal support. The location of center of rotation is affected by both intrinsic and extrinsic factors such as the root length, morphology, periodontal status and the force application point. On the other hand, in this study, we did not aim to focus on very specific condition who's the center of rotation, crown, root and total lengths were given. We aimed to focus on a perspective that if your goal is incisor protrusion for bite opening, you must pay attention to the initial incisor axial inclination. Because, for the same tooth, the same amount angular protrusions are not resulted in the same amount of bite openings. In addition to that, the other goal was presenting of a simple trigonometric calculation as a diagnostic tool for the clinicians.

According to our results, increased initial axial inclination causes much more relative intrusion compared to the lower initial values. For example, when the upper incisor, which has a 90° axial inclination to the true horizontal line, protrudes by 5°, the amount of relative intrusion is 0.0608 mm. However, when the upper incisor, which has a 95° axial inclination to the true horizontal line, protrudes 5°, the amount of relative intrusion increases 3-fold to 0.1824 mm (Table 3). When considering a closer inclination to the normal values, although the differences are reducing, increased initial axial inclinations still cause more relative intrusion compared with the lower initial values (115°→120°, 0.6448 mm; 110°→115°, 0.5344 mm) (Table 3). In the present study, the reason for the presentation of extreme values, such as 90°, 95°, 100°, or 140° and above, is to better understand the issue. Likewise, the accuracy of reverse hypothesis can be proven. In open bite tendency cases, if creating an overbite is intended with the aid of uprighting the incisor, the amount of overbite that will occur will be affected from the initial axial inclination. For instance, when the incisors of two patients

that have the same lengths and different initial axial inclinations are equally uprighted, the amount of overbite will be greater in the patient who has the higher initial axial inclination.

Clinicians are already aware of the bite opening or closing as a result of incisor protrusion or retrusion. However, it was our intention to show that the initial axial inclination can affect the amount of bite opening or closing with the aid of trigonometry. In this study, the protrusion of an average sized upper incisor was simulated. Although the same calculations can be performed for lower incisors, the excessive calculations were considered to be a little boring for readers. The method described in this study can be used for upper and lower incisors, in both protrusion and retrusion movements.

Protrusion is one of the most effective methods in bite opening. However, this is not a real intrusion and it is a relative condition that contributes to the opening of the bite. Genuine intrusion, described by Burstone, is an apical movement of the geometric center of the tooth in respect to the occlusal plane or a plane based on the long axis of the tooth. The condition, which results from labial tipping of the incisors, has been named "pseudo-intrusion" [5]. Because almost all intrusion mechanics cause incisor protrusion, and the condition of pure intrusion occurs rarely, in the present study the term "relative intrusion" was preferred in order to express this condition instead of pseudo-intrusion. In the light of this concept, the term "relative extrusion" can be used for defining the effect of incisor uprighting on overbite.

Although trigonometry has been neglected until now in the field of orthodontics, it can be utilized in many orthodontic topics. In many cases in which angular movement occurs, trigonometry can be a useful tool for determining the amount of movement in different directions. For instance, when rotational mandibular movement is planned as a part of orthognathic surgery, the initial mandibular plane angle is significant with regard to chin prominence. In two other cases, which have equal mandibular corpus length and are intended to have the same amount of angular mandibular movement in a counterclockwise direction, the initial mandibular plane angle affects the chin prominence. In the case of a lower mandibular plane, chin prominence—as a result of the counterclockwise rotation of the mandible—will be less than a case that has a higher mandibular plane. This is because, in the first case, less sagittal movement in the mandibular anterior projection occurs as a result of surgery. When considering the orthodontic and orthognathic surgery applications, the number of samples in which trigonometry is utilized can increase, such as the gingival display in maxillary advancement.

## 5. Conclusions

Initial incisor axial inclination affects the influence of protrusion on bite opening. In addition, it was suggested "relative intrusion" term for defining bite opening as a result of protrusion, and "relative extrusion" term for vice versa.

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